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ICE PRESSURE AGAINST DAMS: EXPERIMENTAL INVESTIGATIONS BY THE BUREAU OF RECLAMATION

By G. E. Monfore

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EXPLANATORY STATEMENT

In recent years a number of changes have been observed in the concepts applied to the design of masonry dams. The work that leads to such changes is being done by different men, different organizations, and often in different countries. Under a Power Division chairman, a Joint Committee on Masonry Dams was formed in 1938, with representatives from the Construction, Irrigation, Power, Soil Mechanics and Foundations, Structural, and Waterways divisions, to make that widespread experience readily available through the medium of the Society's technical publications.

The Subcommittee on Ice Pressure, formed in 1947, has conducted a vigorous search for basic information on the pressure that ice exerts against dams—in Switzerland, Norway, Sweden, and Canada, as well as the United States.

Three papers (*Proceedings-Separate, Nos. 160, 161, and 162*) prepared under the sponsorship of the Subcommittee, are presented to encourage the assembly of facts and figures on this important subject. Each paper is open to discussion, within its scope, independently of the remaining two. When the discussion is closed and the authors' rebuttals have been presented, the group will be collated as a single symposium paper in *Transactions*, from which reprints will be available. With the completion of this work, the Subcommittee recommends that:

(1) Interested organizations be encouraged to develop their work in connection with ice pressure against dams, and urged to maintain mutual liaison in this field;

(2) The present Symposium be published by the Society as a statement of the present state of knowledge regarding ice pressure against dams, and as a basis for discussion; and

(3) The Society, possibly through the Power Division, should reconsider the matter in three or four years' time when the results of further investigations have become generally available.

The *Transactions* printing will include a final report, presented as a "Foreword" to the Symposium.—Ed.

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PAPERS

ICE PRESSURE AGAINST DAMS: EXPERIMENTAL INVESTIGATIONS BY THE BUREAU OF RECLAMATION

By G. E. Monfore[1]

Synopsis

The pressures produced by the thermal expansion of thick ice sheets are considered by the Bureau of Reclamation (USBR), United States Department of the Interior, in the designs of some dams and accessory structures. The USBR conducted experimental investigations of ice pressure from 1946 to 1951. These investigations included field studies at several reservoirs located in the mountains of Colorado and laboratory studies performed in the Engineering Laboratories of the USBR in Denver, Colo. The purpose of this paper is to summarize the more important findings of these field and laboratory investigations.

Field Investigations

The primary aim of the field investigation was to measure directly the pressures that develop in natural ice sheets. However, before pressures could be measured, it was necessary to design and perfect suitable gages. A gage diameter of 2.5 in. was chosen because it was small enough to allow the determination of a pressure gradient between the top and the bottom of an ice sheet. The pressure resulting from an increase in temperature of a restrained ice sheet would be expected to be greatest at the top surface where the temperature change is greatest, and zero on the bottom surface where the temperature remains constant at 32° F. Proper values for gage modulus and pressure range were known only after considerable experimentation in both the laboratory and field.

Note.—Written comments are invited for publication; the last discussion should be submitted by June 1, 1953.

[1] Research Engr., Bureau of Reclamation, U. S. Dept. of the Interior, Denver, Colo.

Electric Resistance Pressure Gage[2].—One type of pressure gage that proved successful is shown in Fig. 1. The gage consisted of two steel plates between which was a strain-sensitive element. As the gage was subjected to external pressure, the separation between the plates decreased by a minute amount. This slight movement was measured by determining the change in electrical resistance of a grid of fine wire that was stretched as the gage deformed.

The strain-sensitive grid consisted of 0.001-in. Advance wire wound on two small glass rods that were contained in a recess between the gage plates. The

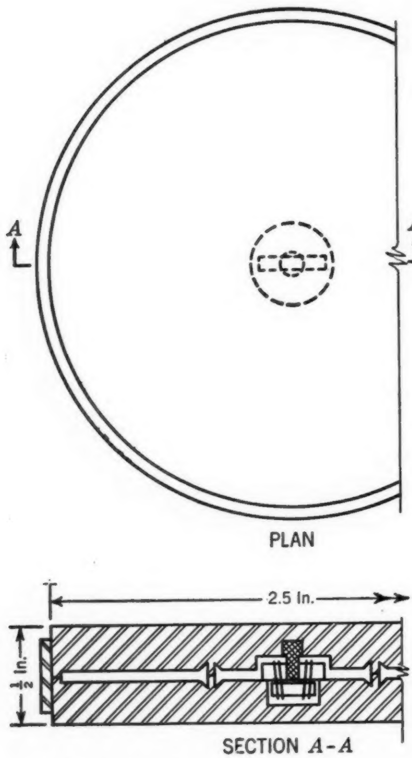

Fig. 1.—Electric Pressure Gage

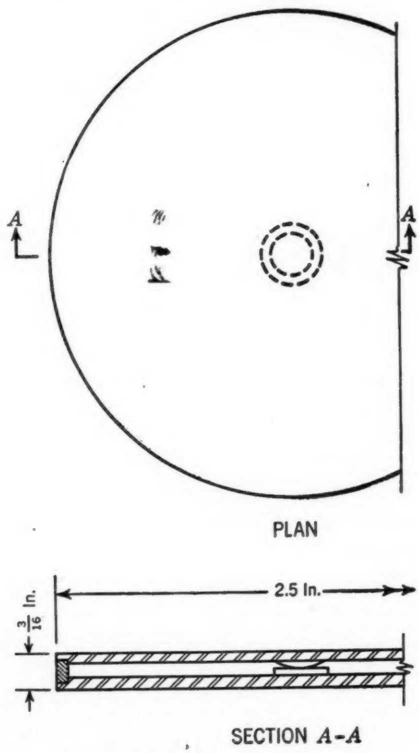

Fig. 2.—Indentor Pressure Gage

longer glass rod was supported on the edge of the recess, but the shorter rod was held only by the wire. When the gage was deformed, the shorter glass rod was forced away from the longer rod and thus the wire was stretched. The resistance of the grid was adjusted to about 120 ohms in order that changes in resistance could be measured with commercial bridges.

The gage was compensated for temperature change by employing a "dummy" grid (not shown in Fig. 1) which was similar to the active grid except that it was not subjected to strain as a result of gage deformation. The grids

[2] "Ice Pressure Measurements at Eleven Mile Canon Reservoir During January 1949," by G. E. Monfore, *Report No. SP-21*, Structural Research Lab., U. S. Bureau of Reclamation, Denver, Colo., April 28, 1949.

were connected so as to form two adjacent arms of a Wheatstone bridge circuit. Resistance changes caused by temperature variation alone do not affect the balance of the bridge if the active and dummy grids are exactly matched.

It has been shown[3] that the modulus of a gage, which is defined as the pressure applied to the gage divided by the change in thickness per unit thickness at the center of the gage, should be equal to, or slightly higher than, the modulus of the material in which the gage is embedded. Although ice is plastic, and hence the theory of elasticity does not apply strictly, its behavior will approach that of an elastic solid for short time changes. If rapid changes in ice pressure are to be measured, the gage modulus must be close to the effective ice modulus which has been reported[4] to be 1.4×10^6 lb per sq in. The modulus of the electric resistance pressure gage was 2.0×10^6 lb per sq in.

Indentor Pressure Gage.—Another type of pressure gage which was used for measuring ice pressure in the field is shown in Fig. 2. This "indentor" gage, although registering only maximum pressure, was considerably simpler to build than the electric gage, and required no attention during the period it was installed in the field.

The indentor gage consisted of two circular steel plates held 0.0625 in. apart by a steel ring 2.5 in. in diameter. The sensitive element was composed of an indentor and a target. The indentor, a segment of a 0.375-in. hardened steel ball, was soldered to one plate, and the polished target of unhardened drill rod was soldered to the other plate. The dimensions were such that the indentor and target were in contact when the gage was assembled. As pressure was applied to the gage, the indentor was driven into the target making a permanent impression. The diameters of the impressions were measured with a microscope fitted with a filar eyepiece.

The modulus of the indentor gage was 0.03×10^6 lb per sq in. From the previous explanation concerning the gage modulus, it is apparent that the indentor gage was not suitable for measuring rapidly changing ice pressures. The gage was satisfactory, however, for recording seasonal maximums. The field investigation showed that pressures develop nearly every day during the ice season and may be operative for several hours each day; hence, there was considerable time for the equalization of the stress in the ice and in the gage by plastic flow in the ice. In order to take full advantage of this plastic flow there must be no tendency for the gage plates to return to a position of zero stress during periods of zero ice pressure. For this reason, the plates were adjusted so that, were the indentor and target removed, the space between the plates would be about 0.002 in. less than the combined thickness of the indentor and target. Thus there was an initial outward bending of the plates, which kept the indentor and target in contact at all times.

Pressure Measurements.—Before either the electric gages or the indentor gages were installed in the field, they were embedded in thin mortar panels as indicated in Fig. 3. The panel was installed with the three gages in a vertical line. The gages were spaced so as to be symmetric in an 18-in. ice sheet. The

[3] "An Analysis of the Stress Distributions in and near Stress Gages Embedded in Elastic Solids," by G. E. Monfore, *Report No. SP-26*, Structural Research Lab., U. S. Bureau of Reclamation, Denver, Colo., June 26, 1950.

[4] "Sonic Determination of the Elastic Properties of Ice," by T. D. Northwood, *Canadian Journal of Research*, Vol. 25, Section A, March, 1947, p. 88.

center of the top gage was placed 2.5 in. below the top ice surface, the center of the bottom gage was 2.5 in. above the lower ice surface, and the middle gage was at the center of the ice sheet. The mortar panels were installed at the reservoirs after the ice had reached sufficient thickness for safety. Holes were cut through the sheet and the panels were supported by wires at the proper height. The panels were not removed until the ice had melted in the spring.

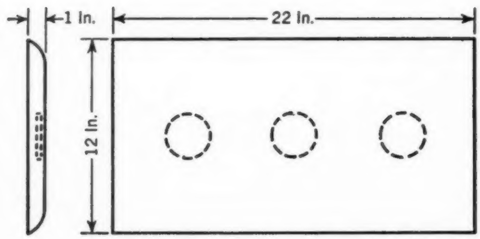

FIG. 3.—POSITIONS OF PRESSURE GAGES IN MORTAR PANEL

Ice pressures were measured with six of the electric gages at Eleven Mile Canon Reservoir[2] during a period of two weeks in January, 1949. This reservoir is part of Denver's municipal water system, and is located in central Colorado at El. 8600 ft, a latitude of 38° 54′ N, and a longitude of 105° 28′ W. The dam is a concrete arch 445 ft long and 125 ft high. The shores of the reservoir in the vicinity of the dam are very steep and rocky. A typical gage location is shown in Fig. 4.

The gages were read four or five times during daylight hours every day for two weeks. In general, high pressures occurred during relatively warm weather following periods of cold weather. The highest pressure recorded was 150 lb per sq in. at 12:45 p.m. on January 22, by a top gage. At the same time, the middle gage showed 50 lb per sq in., and the bottom gage, 0 lb per sq in. By 3 p.m., the pressure on the top gage had decreased to 100 lb per sq in., the pressure on the middle gage had increased to its peak value of 80 lb per sq in., and the pressure on the bottom gage was 5 lb per sq in. The average pressure was highest at 12:45 p.m., when it was $(150 + 50 + 0)/3 = 67$ lb per sq in. Because the ice sheet was 18 in. thick at the time, the thrust was calculated as $67 \times 12 \times 18 =$ 14 kips per lin ft. The highest average pressure, 74 lb per sq in., occurred on January 20. The thrust corresponding to this average pressure was 16 kips per lin ft.

On several occasions, pressures were measured when there was a strong wind, and at each of these times rapid variations in the readings were noted. Undoubtedly, these variations were caused by the wind, which was very gusty in Eleven Mile Canon. The magnitudes of the variations were estimated by observing the bridge for a period of several minutes and adjusting it for both the high and low points of fluctuation. The greatest variation from the mean pressure was 3 kips per lin ft for a wind that reached gusts of perhaps 30 miles per hr.

During one severe cold period, numerous cracks that did not close during the day were observed in the top of the ice sheet. Apparently the ice was cold enough to freeze the water that entered the contraction cracks from beneath the sheet before the cracks were filled. During this cold period the pressures were generally low, and the highest pressures were recorded, not by the top gage, but by the middle gage.

Several indentor gages were installed at Eleven Mile Canon Reservoir[5,6] during each of the winters of 1947–1948, 1948–1949, and 1949–1950. As with the electric gages, the indentor gages indicated that the highest pressures usually occurred at the top of the ice sheet and the lowest at the bottom. There was some uncertainty in translating the pressure readings of the indentor gages to values of thrust because the impressions of the top, middle, and bottom gages may have occurred at different times. Thrusts were usually calculated by taking the average pressure recorded by the top, middle, and bottom gages times the area of 1 lin ft. The seasonal maximum was 16 kips per lin ft in 1947–1948, 14 kips per lin ft in 1948–1949, and 20 kips per lin ft in 1949–1950.

Fig. 4.—Pressure Gage Installation at Eleven Mile Canon Reservoir

It is reasonable to expect that the shores of a reservoir would influence the thrust developed in an ice sheet. If the shores yield under load, the thrust should be relieved accordingly. If the ice sheet can slide up a gently sloping shore, the thrust should also be relieved.

Tests made in the winter of 1950–1951 at several reservoirs[7] offering a considerable variety of shore conditions seemed to confirm this expectation.

[5] "Ice Pressure Measurements by Means of Indentor Gages at Eleven Mile Canon Reservoir, Winters of 1947-48 and 1948-49," by G. E. Monfore, *Report No. SP-22*, Structural Research Lab., U. S. Bureau of Reclamation, Denver, Colo., August 4, 1949.

[6] "Ice Pressure Measurements by Means of Indentor Gages at Eleven Mile Canon Reservoir for the Winter of 1949-50," by G. E. Monfore, *Report No. SP-28*, Structural Research Lab., U. S. Bureau of Reclamation, Denver, Colo., October 16, 1950.

[7] "Ice Pressure Measurements by Means of Indentor Gages for the Winter of 1950-51," by G. E. Monfore, *Report No. SP-33*, Structural Research Lab., U. S. Bureau of Reclamation, Denver, Colo. December 12, 1951.

Indentor gages were installed at Antero (El. 9000), Evergreen (El. 7050), Shadow Mountain (El. 8370), and Tarryall (El. 9000) reservoirs. All are in Colorado. The highest thrust at Antero Reservoir was 3.6 kips per lin ft and at Shadow Mountain was 5.8 kips per lin ft. Both reservoirs have flat shores. The greatest thrust at Evergreen Reservoir, which has moderately steep shores, was 9.4 kips per lin ft. The highest thrust for the winter, 17 kips per lin ft, was recorded at Tarryall Reservoir which has steep, rocky shores in the vicinity of the dam where the gages were located. These results indicate that the type of shore can have an important effect on the thrust developed.

In seeming contradiction to these results, however, were those obtained at Eleven Mile Canon Reservoir. The thrust recorded during the winter of 1949–1950 near a shallow sandy beach that was located a mile above the dam was nearly the same as that measured near vertical rock faces in the vicinity of the dam.[6] In this case, the gently sloping shore did not relieve ice pressure.

The explanation for these apparently contradictory results may be the strength of the bond between an ice sheet and a flat shore. The location of the shore and its orientation relative to surrounding surface features and to solar radiation could influence the bond strength and thereby the maximum thrust.

Other Measurements.—Ice temperatures were measured at Eleven Mile Canon Reservoir[2] during January, 1949, at depths in the ice sheet of 1 in., 2 in., 4 in., 8 in., and 16 in. One set of iron-constantan thermocouples was put in ice that was clear of snow, and another set was placed in ice that had a 5-in. snow cover. Four or five readings were made during daylight hours at each thermocouple. The temperature at the top of the sheet began increasing at about 9 a.m. and increased until about 3 p.m. At lower levels in the ice sheet the temperature began increasing at a later time and continued until later in the afternoon. The effect of snow in insulating the ice was remarkable. The temperature at the top of the clear ice increased an average of 13.5° F from the low to the high during a nine-day period, while the temperature at the top of the ice with a 5-in. snow cover increased an average of only 3.3° F. The lowest temperature recorded at the 1-in. depth in clear ice was − 7° F, and the greatest rate of rise in ice temperature was 7° F per hr. The effect of these ice temperature changes on pressure will be further discussed subsequently under the heading, "Calculation of Ice Thrust."

Air temperatures were recorded continuously at Eleven Mile Canon Reservoir during the winter of 1948–1949, and daily maximum and minimum temperatures were read for the other winters of test. The extremes for the two months of January and Februrary were as follows:

Year	Minimum temperature, in degrees Fahrenheit	Maximum temperature, in degrees Fahrenheit
1948	−45	52
1949	−29	51
1950	−16	54
1951	−30	60

The only air temperature records for the other reservoirs where pressure gages were installed in the winter of 1950–1951 were maximum and minimum read-

ings. The extremes for the two months of January and Februrary were the following: At Antero Reservoir, a minimum of $-50°$ F and a maximum of 66° F; at Evergreen Reservoir, a minimum of $-32°$ F and a maximum of 72° F; and at Shadow Mountain Reservoir, a minimum of $-41°$ F and a maximum of 51° F. No records were available for Tarryall Reservoir, but it is located only 20 miles from Eleven Mile Canon Reservoir and has approximately the same elevation.

The absorption of solar radiation by an ice sheet is an important factor influencing the temperature of the ice. The intensity of solar radiation was measured at Eleven Mile Canon Reservoir during the winter of 1946–1947, and some tests[2] were made in January, 1949, to determine the amount of energy absorbed in the ice. The absorption data were widely scattered but indicated that the absorption in ice is nearly the same as that in water.[8]

The thickness of the ice sheet at Eleven Mile Canon Reservoir was measured several times each winter. The maximum thickness was 24 in. in 1948, and 20 in. in 1949 and 1950. The ice reached a thickness of 19 in. at Evergreen Reservoir in 1951.

Laboratory Investigation[9]

The purpose of the laboratory investigation was to supplement the field program in determining the magnitude of thrust and to provide basic information on the action of ice under changing temperature and pressure. Ice cylinders were subjected to rising temperature, and the pressures necessary to prevent axial expansion were determined. This is basically the procedure used by Ernest Brown and George C. Clarke,[10] M. ASCE, in their laboratory investigation of ice pressure. However, the apparatus used in the present investigation was entirely different, and the results were significantly different in magnitude.

Samples.—Cylinders 4 in. in diameter were cut from cakes of field ice in such a manner that the axes of the cylinders corresponded to the horizontal direction in the original ice sheet. Loads applied to the ends of the cylinders were thus in the same direction relative to the crystal structure as thrusts that develop in the field. The cylinders were cut to a length of 4 in., and bearing blocks consisting of brass plates $\frac{1}{4}$ in. thick and plaster of Paris cylinders 4 in. by 4 in. were frozen to each end. This particular construction of the bearing blocks was found to provide a uniform temperature distribution within the ice sample.

Apparatus.—The insulated chamber in which the pressure tests were conducted was 9 in. in diameter by 12 in. high. The ice sample with its bearing block assembly were placed between masonite plugs, 6 in. in diameter by 4 in. long, located at the top and bottom of the test chamber. These masonite plugs served not only as insulators for the ends of the test chamber, but also as part of the loading system. The construction of the test chamber and the

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[8] "Properties of Ordinary Water-Substance," by N. E. Dorsey, Reinhold Publishing Corp., New York, N. Y., 1940, p. 333.

[9] "Laboratory Investigation of Ice Pressure," by G. E. Monfore, *Report No. SP-31*, Structural Research Lab., U. S. Bureau of Reclamation, Denver, Colo., October 8, 1951.

[10] "Ice Thrust in Connection with Hydro-Electric Plant Design," by Ernest Brown and George C. Clarke, *The Engineering Journal*, January, 1932, pp. 18–25.

position of the ice sample are shown in Fig. 5. Loads were applied to the sample by a small hydraulic ram connected to a hand pump that was equipped with a pressure gage.

The temperature of the ice sample was controlled by circulating air of the desired temperature through the test chamber. The air was furnished by an air temperature controller consisting principally of a centrifugal fan, an open mesh wire basket containing dry ice, and an electric heating element controlled by a variable transformer. The various components were housed in a large insulated box to which the test chamber was attached. A damper in the controller made it possible to circulate all the air through the dry ice basket, to by-pass the dry ice and circulate the air over the heating element, or to circulate a portion of the air through the dry ice and the remainder over the heating element.

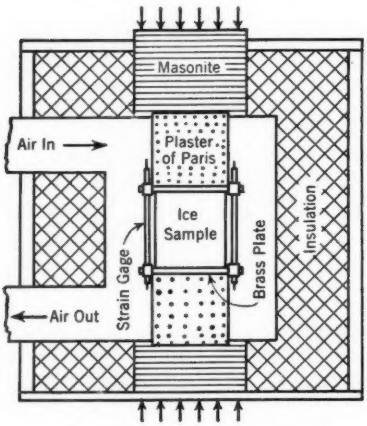

FIG. 5.—TEST CHAMBER

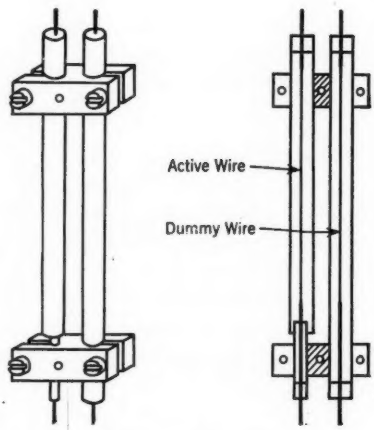

FIG. 6.—STRAIN GAGE

The temperature of the air stream through the test chamber was measured with 36-gage copper-constantan thermocouples, and ice temperatures were measured with similar thermocouples embedded in the ice. The values of thermal electromotive force were measured with a manual potentiometer that was sensitive to a change of 0.001 mv. This corresponds to approximately 0.05° F for copper-constantan thermocouples.

The requirements for a gage needed to insure a constant length for an ice sample during a temperature rise may be estimated by assuming the sample to be elastic. If the modulus of ice is taken to be 1.4×10^6 lb per sq in., an error in length of the sample corresponding to a strain of one part in a million would result in an error of 1.4 lb per sq in. in the stress in the ice. Because a finite time was required to adjust the load on the ice, and because ice is a plastic solid, the actual error would be less than that calculated on the assumption of an elastic sample.

A gage that was sensitive to a strain of one part in a million was devised and perfected in the laboratories of the USBR. A drawing of this unbonded elastic wire gage is shown in Fig. 6. The gage consisted of an active wire

that was sensitive to length changes in the specimen, and a dummy wire that was included in the bridge circuit for temperature compensation. The dummy tube also served as a holder for the gage when it was not in use. The gage was fastened to the brass plates at each end of the ice sample. The clamping screw that held the dummy tube to the upper gage block was loosened and the dummy tube was lowered until it was free from the upper block. The active wire was then stretched to about one half of its elastic limit and clamped. Any change in length of the ice made the same change in length of the active wire. It was thus possible to keep the length of the ice sample constant to within one part in a million.

Test Procedure.—With the strain gages attached and the sample in place in the test chamber, the air temperature was brought to the desired initial temperature and held constant until equilibrium was established. In order to obtain a given linear rate of increase in the ice temperature, it was necessary to increase the air temperature at a much greater rate for about the first 15 min. As the temperature of the ice increased at the chosen rate, the load on the sample was increased so as to maintain the ice at its initial length. The temperatures were read and the load on the sample was regulated every 5 min for the first 30 min of test; thereafter, the temperatures were read every 15 min and the load was adjusted twice every 15 min.

The ice temperatures and pressures for a typical test are shown by the curves of Fig. 7. In this particular test, the ice temperature increased from an initial temperature of $-10.5°$ F at a uniform rate of 5° F per hr. The pressure increase was fairly linear for the first part of the test, then curved to a maximum, and decreased. Many tests were performed under various temperature conditions, and invariably the pressure increased to a maximum and then decreased.

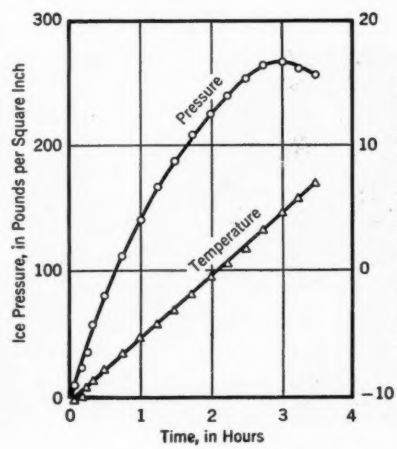

Fig. 7.—Typical Ice Pressure Test

Results.—Check tests were run on several samples to determine the reproducibility of the apparatus and technique. The average deviation in the maximum pressures for duplicate tests on a given sample was 6%. This was considered satisfactory precision.

Much larger variations, an average of 25%, were found in the maximum pressures reached by different ice samples tested under the same conditions. Similarly large variations in the physical properties of ice have been reported by other investigators.

A possible explanation for such large variations in otherwise apparently identical samples might lie in the arrangement of the crystals. Therefore, it was decided to examine the crystal structure of some of the samples that had

been included in the pressure tests. The whole samples were placed between crossed polaroids and the crystal arrangements were studied. This was later done with sections ½ in. thick, also. Photographs were made of those orientations which seemed to be important. Fig. 8 shows cross sections of the crystals in two samples. Sample A showed poorly defined crystals and no sharp division lines. The maximum pressure obtained with this sample was 326 lb per sq in. when tested from an initial temperature of −10° F with a temperature rise in the ice of 5° F per hr. Sample B, although having poorly defined crystals, did show a sharp division line extending at roughly 45° to the load axis. This sample reached a maximum pressure of only 145 lb per sq in. under the same test conditions as those used for sample A. The indications would seem to be that slippage along the sharp division surface caused the lower pressure in sample B. A study of the other samples revealed similar situations;

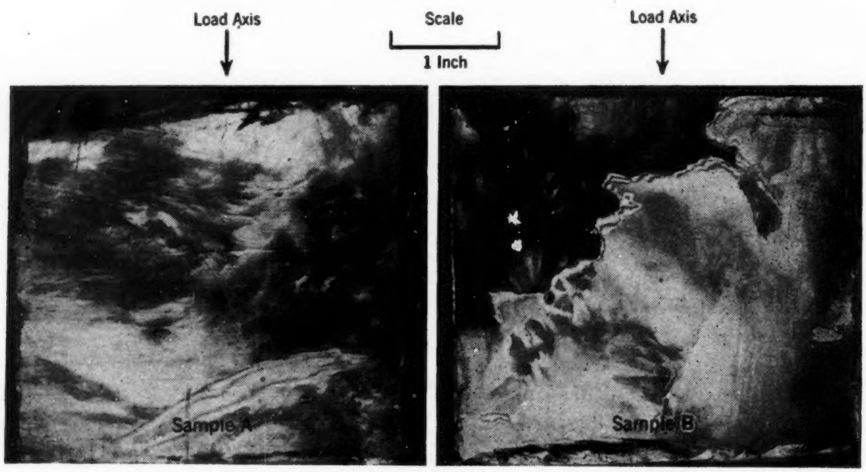

Fig. 8.—Crystal Structure of a Strong Sample and a Weak Sample

that is, the samples that exerted low maximum pressures contained crystal surfaces or other sharp divisions along which slippage might occur.

Pressure tests were made with initial temperatures of −30° F, −10° F, and +10° F, and with rates of temperature rise in the ice of 2° F per hr, 5° F per hr, 10° F per hr, and 15° F per hr. Curves similar to those of Fig. 7 were obtained for each test. For a given rate of temperature rise, the maximum pressure that the sample reached increased as the initial temperature decreased; and, for a given initial temperature, the maximum pressure increased as the rate of temperature rise increased. The times required for the maximum pressures to develop, and the relationships between maximum pressure and rate of temperature rise for several initial temperatures are shown by the series of curves in Fig. 9. These curves represent the averages obtained from more than 100 tests performed on 24 ice samples.

Computation of Ice Thrust

The laboratory data can be used to calculate ice thrusts that may develop in the field, if information on ice sheet temperatures is available. Analytical methods for computing temperatures in ice sheets for various assumed weather conditions have been developed by Edwin Rose[11] and Frank W. Taylor.[12] The maximum pressures corresponding to such computed ice temperatures can be read from the curves of Fig. 9.

Ice pressures can be computed readily for the ice temperatures that were measured at Eleven Mile Canon Reservoir in January, 1949. Furthermore, the simultaneous measurement of ice pressures with the electric gages permits

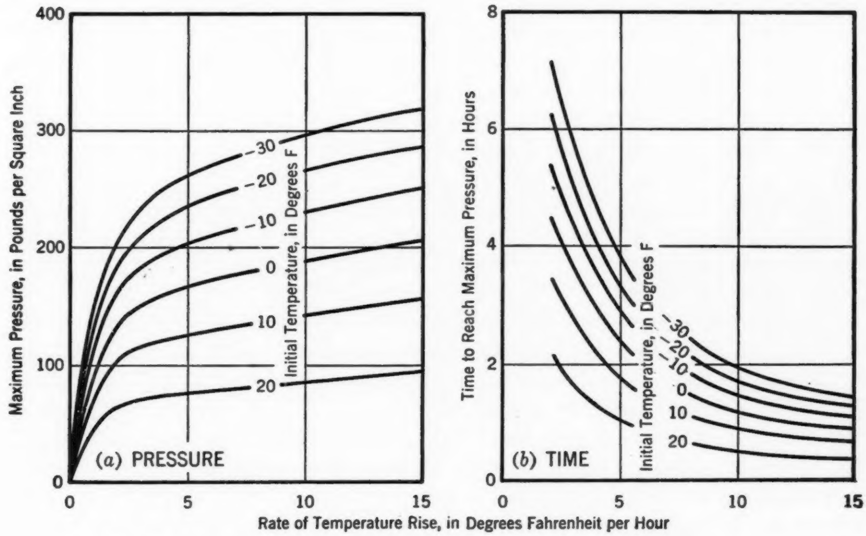

Fig. 9.—Maximum Ice Pressure and Time of Temperature Rise Related to Rate of Ice Temperature Rise

a direct comparison of measured and calculated thrusts. Because several factors may interfere with the development of thrust, there are probably few days during a winter when the highest possible thrust is reached. Three days that were most nearly free of interference were selected for study. Ice temperatures at a depth of 1 in. were used to calculate pressures at the top of the ice sheet, and temperatures at a depth of 8 in. were used to calculate pressures in the middle of the sheet.

The steps in computing thrusts from ice temperatures can best be illustrated by considering one case in detail. The ice at the top of the sheet reached a minimum temperature of 14° F on the morning of January 22, 1949. The ice temperature began increasing at about 9 a.m. and continued increasing

[11] "Thrust Exerted by Expanding Ice Sheet," by Edwin Rose, *Transactions*, ASCE, Vol. 112, 1947, p. 895.

[12] "Temperature Changes in an Ice Sheet, with the Lower Surface in Contact with Water at Freezing Temperature, for Various Conditions of Exposure on Upper Surface," by Frank W. Taylor, Memorandum of November 29, 1945, U. S. Bureau of Reclamation, Denver, Colo.

for about 6 hr. The greater part of the rise occurred, however, in 4 hr. The rate of rise that would be applicable for computing thrust was estimated as 3.6° F per hr. The maximum pressure resulting from a temperature rise of 3.6° F per hr from an initial temperature of 14° F is found to be 100 lb per sq in. from the curves of Fig. 9(*a*), and the time required to reach this maximum pressure is 2 hr from Fig. 9(*b*). The minimum temperature measured in the ice sheet at a depth of 8 in. on the morning of January 22 was 19° F and the rate of rise for this point was 1.3° F per hr. The maximum pressure from Fig. 9(*a*) for an initial temperature of 19° F and a rise of 1.3° F per hr is seen to be 55 lb per sq in. It is estimated from Fig. 9(*b*) that approximately 3 hr were required to develop the maximum pressure under these conditions. The temperature records showed that over 6 hr were available. The calculated ice pressure at the bottom of the sheet is zero because the ice temperature remained constant at 32° F. The average pressure through the sheet was taken as simply $100 + 55 + 0$ divided by three which equals 52 lb per sq in. The maximum thrust was then $52 \times 12 \times 18 = 11$ kips per lin ft for the 18-in. ice sheet. As previously mentioned, the thrust measured with the electric gages on January 22 was 14 kips per lin ft.

The thrust computed in a similar fashion from ice temperatures for January 14 was 6 kips per lin ft and that for January 21 was 13 kips per lin ft. The thrusts measured with the pressure gages were 6 kips per lin ft and 12 kips per lin ft, respectively.

Because the reservoir in the vicinity of the pressure gages was nearly free of snow, the thrusts computed here were correctly based on temperatures measured in clear ice. Temperatures were also measured in ice that had a 5-in. snow cover. The thrusts that might develop in a reservoir completely covered by 5 in. of snow may be calculated from these temperature readings. The thrust thus computed for January 14 was 1 kip per lin ft, that for January 21 was 3 kips per lin ft, and that for January 22 was 3 kips per lin ft. These thrusts are to be compared to thrusts of 6 kips per lin ft, 13 kips per lin ft, and 11 kips per lin ft, which were calculated for ice free of snow.

The most rapid ice temperature rises recorded at Eleven Mile Canon Reservoir were 7° F per hr at a depth of 1 in. and 1.8° F per hr at a depth of 8 in. The lowest recorded ice temperatures, which were not recorded on the same day as the most rapid rises, were −7° F at a depth of 1 in. and 9° F at a depth of 8 in. It was then assumed that the most rapid rises could occur from the lowest ice temperatures. The maximum thrust calculated for these severe conditions was 22 kips per lin ft.

Conclusions

The highest seasonal thrusts measured with indentor gages at Eleven Mile Canon Reservoir for three winters ranged from 14 kips per lin ft to 20 kips per lin ft. Because the shores of the reservoir where the measurements were made offer as severe restraint to an ice sheet as any to be found, these ice thrusts probably represent maximum values for the ice thickness and weather conditions prevailing during the tests. The results obtained with the indentor gages were subject to some uncertainty, but they were in good agreement with the

highest thrust of 16 kips per lin ft which was measured with the electric gages during January, 1949.

These values of thrust were further substantiated by the laboratory investigation. This investigation indicated that the pressure that can develop in a restrained ice sample as the result of a temperature rise is limited by plastic flow. The maximum pressure depends on the initial temperature of the ice and on the rate at which the temperature is increased. The laboratory data were used to calculate the thrusts corresponding to ice temperatures measured at Eleven Mile Canon Reservoir. The thrusts thus calculated were in good agreement with the thrusts measured directly by the electric pressure gages.

The maximum thrusts that might develop in other locations could be calculated from the laboratory data, provided information on ice temperatures becomes available. Temperature measurements in ice sheets at reservoirs of various latitudes and elevations, and during periods extending over several years, would be necessary.

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